(Applied Math. of TEAM)

Given a region $\Omega \subset \mathbb{R}^3$, please deduce the partial differential equations of the fluid movement according to the conservation law of the mass and momentum.

Assume $\rho = \rho(t, x)$ is the density, $u = (u_1, u_2, u_3)(u_i = u_i(t, x))$ is the velocity(speed) of the fluid, and $p = p(t, x)$ is the pressure, where $x = (x_1, x_2, x_3)$ is the space variables.

Question 1: Given two different expression of the change of the mass in $\Omega$ of the fluid with the density $\rho(t, x)$ within the time interval $[t_1, t_2]$.

Question 2: Please deduce the partial differential equations of $\rho, u$ according to the conservation of the mass in $\Omega$.

Question 3: Assume there is no outside force acting on the fluid in $\Omega$, please give two different expression of the change of the momentum of the fluid with the density $\rho(t, x)$ and the velocity $u(t, x)$ in $\Omega$ within the time interval $[t_1, t_2]$.

Question 4: According to the conservation of the momentum of the fluid in $\Omega$, deduce the partial differential equation of $\rho, u, p$.